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INCREASING & DECREASING FUNCTIONS AND MAXIMA & MINIMA

STRICTLY INCREASING FUNCTION :

A function $f(x)$ is said to be a strictly increasing function on (a, b) if

$$x_1 < x_2 \Rightarrow f(x_1) < f(x_2) \text{ for all } x_1, x_2 \in (a, b)$$

Strictly Decreasing Function : A function $f(x)$ is said to be a strictly decreasing function on (a, b) if

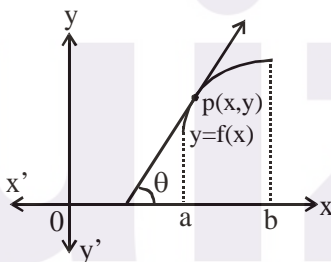
$$x_1 < x_2 \Rightarrow f(x_1) > f(x_2) \text{ for all } x_1, x_2 \in (a, b)$$

By an increasing or a decreasing function we shall mean a strictly increasing or a strictly decreasing function.

Monotonic Function : A function $f(x)$ is said to be monotonic on an interval (a, b) . it is either increasing or decreasing on (a, b)

Definiton : A function $f(x)$ is said to be increasing on $[a, b]$ if it is increasing on (a, b) and it is also increasing at $x = a$ and $x = b$.

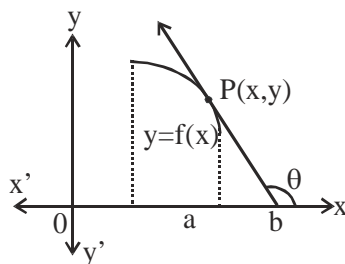
Necessary Condition : We observe that if $f(x)$ is an increasing function on (a, b) then tangent at every point on the curve $y = f(x)$ makes an acute angle θ with the positive direction of x -axis.



$$\therefore \tan \theta > 0 \Rightarrow \frac{dy}{dx} > 0 \text{ or } f'(x) > 0 \text{ for all } x \in (a, b)$$

If $f(x)$ is a decreasing function on (a, b) , then tangent at every point on the curve $y = f(x)$ makes an obtuse angle θ with the positive direction of x -axis.

$$\therefore \tan \theta < 0 \Rightarrow \frac{dy}{dx} < 0 \text{ or } f'(x) < 0 \text{ for all } x \in (a, b)$$



SUFFICIENT CONDITION

THEOREM : Let f be a differentiable real function defined on an open interval (a, b)

- (i) If $f'(x) > 0$ for all $x \in (a, b)$ then $f(x)$ is increasing on (a, b)
- (ii) If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b) .

Properties of Monotonic Function :

- (i) If $f(x)$ is strictly increasing function on an interval $[a, b]$, then f^{-1} exists and it is also a strictly increasing function.
- (ii) If $f(x)$ is strictly increasing function on an interval $[a, b]$ such that it is continuous, then f^{-1} is continuous on $[f(a), f(b)]$.
- (iii) If $f(x)$ is continuous on $[a, b]$ such that $f'(c) \geq 0$ ($f'(c) > 0$) for each $c \in (a, b)$, then $f(x)$ is monotonically (strictly) increasing function on $[a, b]$
- (iv) If $f(x)$ and $g(x)$ are monotonically (or strictly) increasing (or decreasing) functions on $[a, b]$, then $\text{gof}(x)$ is a monotonically (or strictly) increasing function on $[a, b]$
- (v) If one of the two functions $f(x)$ and $g(x)$ is strictly (or monotonically) increasing and other a strictly (monotonically) increasing and other a strictly (monotonically) decreasing, then $\text{gof}(x)$ is strictly (monotonically) decreasing on $[a, b]$.

MAXIMA AND MINIMA

Let $f(x)$ be a function with domain $D \subset \mathbb{R}$. Then $f(x)$ is said to attain the maximum value at a point $a \in D$ if

$$f(x) \leq f(a) \text{ for all } x \in D$$

In such a case, a is called the point of maximum and $f(a)$ is known as the maximum value or the greatest value.

Local Maximum : A function $f(x)$ is said to attain a local maximum at $x = a$ if there exists a neighbourhood .

$(a - \delta, a + \delta)$ of a such that

$$f(x) < f(a) \text{ for all } x \in (a - \delta, a + \delta), x \neq a$$

or
$$f(x) - f(a) < 0 \text{ for all } x \in (a - \delta, a + \delta), x \neq a$$

In such a case $f(a)$ is called the local maximum value of $f(x)$ at $x = a$.

Local Minimum : A function $f(x)$ is said to attain a local minimum at $x = a$ if there exists a neighbourhood

$(a - \delta, a + \delta)$ of a such that $f(x) > f(a)$ for all $x \in (a - \delta, a + \delta), x \neq a$

or
$$f(x) - f(a) > 0 \text{ for all } x \in (a - \delta, a + \delta), x \neq a$$

The value of the function at $x = a$ i.e. $f(a)$ is called the local minimum value of $f(x)$ at $x = a$

Theorem : A necessary condition for $f(a)$ to be an extreme value of a function $f(x)$ is that $f'(a) = 0$ in case it exists. A function may however attain an extreme value at a point without being derivable thereat. For example, the function $f(x) = |x|$ attains the minimum value at the origin even though it is not derivable at $x = 0$.

Remark : Above condition is only a necessary condition for the point. $x = a$ to be an extreme point. It is not sufficient i.e. $f'(a)$ does not necessarily imply that $x = a$ is an extreme point. For example for the function $f(x) = x^3$, $f'(0) = 0$ but at $x = 0$ the function does not attain an extreme value.

Remark : The value of x for which $f'(x) = 0$ are called stationary values or critical values of x and the corresponding values of $f(x)$ are called stationary or turning values of $f(x)$.

Theorem : (First derivative test for local maximum and minima) Let $f(x)$ be a function differentiable at $x = a$.
Then,

(A) $x = a$ is a point of local maximum of $f(x)$, if

- (i) $f'(a) = 0$ and
- (ii) $f'(x)$ changes sign from positive to negative as x passes through a i.e. $f'(x) > 0$ at every point in the left nbd $(a - \delta, a)$ and $f'(x) < 0$ at every point in the right nbd $(a, a + \delta)$ of a .

(B) $x = a$ is a point of local minimum of $f(x)$, if

- (i) $f'(a) = 0$ and
- (ii) $f'(x)$ changes sign from negative to positive as x passes through a i.e. $f'(x) < 0$ at every point in the left nbd $(a - \delta, a)$ of a and $f'(x) > 0$ at every point in the right nbd $(a, a + \delta)$ of a .

(C) If $f'(a) = 0$ but $f'(x)$ does not change sign, that is $f'(a)$ has the same sign in the complete nbd of a , then a is neither a point of local maximum nor a point of local minimum.

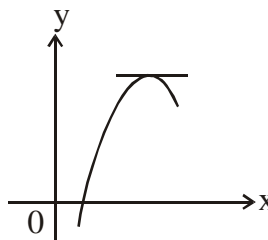
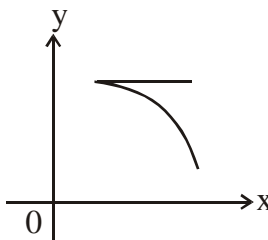
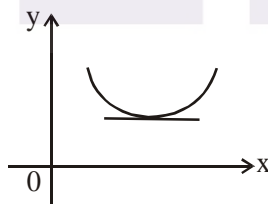
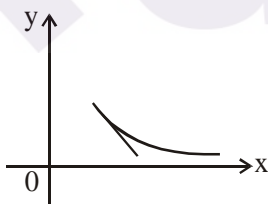
Theorem : (Higher order derivative test). Let f be a differentiable function on an interval I and let c be an interior point of I such that

- (i) $f'(c) = f''(c) = f'''(c) = \dots = f^{n-1}(c) = 0$ and
- (ii) $f^n(c)$ exists and is non-zero

Then,

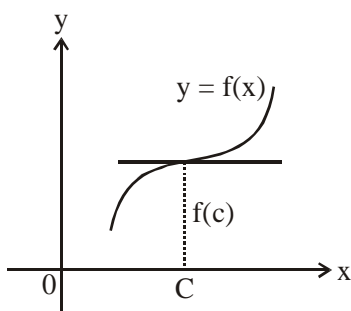
- (a) If n is even and $f^n(c) < 0 \Rightarrow x = c$ is a point of local maximum.
- (b) If n is even and $f^n(c) > 0 \Rightarrow x = c$ is a point of local minimum
- (c) If n is odd $\Rightarrow x = c$ is neither a point of local maximum nor a point of local minimum.

Point of inflection : An arc of a curve $y = f(x)$ is called concave upward if, at each of its points, the arc lies above the tangent at the point. An arc of a curve $y = f(x)$ is called concave downward if, at each of its points, the arc lies below the tangent at the point.



Definition :

A point of inflection is a point at which a curve is changing concave upward to concave downward, or vice-versa.



A curve $y = f(x)$ has one of its points $x = c$ as an inflection point

If $f''(c) = 0$ or is not defined and

If $f''(x)$ changes sign as x increases through $x = c$.

The later condition may be replaced by $f'''(c) \neq 0$ when $f'''(c)$ exists.

Thus, $x = c$ is a point of inflection if $f''(c) = 0$ and $f'''(c) \neq 0$.

Critical point : A point $x = \alpha$ is a critical point of a function $f(x)$ if
 $f'(\alpha) = 0$ or $f'(\alpha)$ does not exist.

Quizrr